Effect of Magnetic Quantization on the Effective Electron Mass in Bismuth

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Z. Naturforsch. **41 a**, 881–882 (1986); received December 17, 1985

An attempt is made to derive an expression for the effective electron mass along the direction of a magnetic field in bismuth under magnetic quantization by using the generalized dispersion relation of the conduction electrons as proposed by Meclure and Choi. It is found that the oscillatory quantum number dependent behaviour of the effective electron mass at the Fermi level in bismuth is solely due to band non-parabolicity and is a characteristic feature of the above model.

The effective mass of the carriers in semiconductors is a very important parameter and has been investigated under various physical conditions [1–4]. Nevertheless, the effective electron mass in bismuth in the presence of a of a quantizing magnetic field has yet to be theoretically worked out for the more difficult case which occurs from the use of the generalized dispersion relation for the conduction electrons in Bi in the absence of any quantization as proposed by MeClure and Choi [5]. This is important since the above model has been stated in the literature [6] to be the most valid model in bismuth and also explains the various experimental results. However, we shall consider the effective electron mass at the Fermi level, since at very low temperatures the electrons at the Fermi surface are the major participants in electron transport.

In the presence of quantizing magnetic field B along z-direction the modified electron energy spectrum (up to the first order) can be expressed [6], neglecting spin-effects, as

$$E=(p-1)\,E_g/2, \tag{1}$$

where the notations are the same as in [6].

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Therefore, the effective electron mass at the Fermi level (E_F) can be written as

$$\begin{aligned} m_n^*(E_F) = m_3\,[(n+\tfrac{1}{2})\,\hbar\,\omega_1(E)\,(2E_g)^{-1} \\ \cdot A^{-2}(E)\,\{E(1+\alpha E)-(n+\tfrac{1}{2})\,\hbar\,\omega(E)-D(E)\} \\ + A^{-1}(E)\,\{1+2\alpha E-(n+\tfrac{1}{2})\,\hbar\,\omega_1(E)-D_1(E)\}]\,|_{E=E_F}\,, \end{aligned} \tag{2}$$

where

$$A(E) \equiv [1-(n+\tfrac{1}{2})\,\hbar\,\omega(E)\,(2E_g)^{-1}]\,,$$

$$D(E) \equiv (n^2+n+1)\,(4E_g)^{-1}\,[\hbar\,\omega(E)]^2, \quad \alpha \equiv 1/E_g\,,$$

$$\omega_1(E) \equiv [2\,\omega(E)]^{-1}\left[\alpha\,e^2 B^2\left(1-\frac{m_2}{m_2'}\right)/m_1\,m_2\right] \quad \text{and}$$

$$D_1(E) \equiv 2\,D(E)\,\omega_1(E)\,\omega(E)\,.$$

For $\alpha \to 0$, (2) takes the form

$$m_n^*(E_F)=m_3\,. \tag{3}$$

Thus for $\alpha \to 0$, the effective Fermi level mass becomes independent of the magnetic field and magnetic quantum number, respectively. The band nonparabolicity can alone explain the oscillations in the effective electron mass in most of the Kane type semiconductors where the effective mass is strictly index independent. The speciality of the MeClure and Choi model is that the same nonparabolicity also explains the quantum number dependent behaviour of the same mass in bismuth.

It appears therefore that for obtaining the dependence of $m_n^*(E_F)$ on $1/B$ for a fixed electron concentration n_0, one requires a relation between n_0 and E_F which in turn can be expressed, at a finite temperature and neglecting small tilt angles, as

$$n_0=[3\,e\,B\,\sqrt{2m_3}/\pi^2\,\hbar^2]\sum_{n=0}^{n_{\max}}[P(E_F)+Q(E_F)]\,, \tag{4}$$

where

$$\begin{aligned} P(E_F) \equiv [A^{-1}(E_F)\,\{E_F(1+\alpha\,E_F) \\ -(n+\tfrac{1}{2})\,\hbar\,\omega(E_F)-D(E_F)\}]^{1/2}\,, \end{aligned}$$

$$\begin{aligned} Q(E_F) \equiv \sum_{r=1}^{S}\left[\frac{d^{2r}}{dE_F^{2r}}\{P(E_F)\}\right] \\ \times[2\,(1-2^{1-2r})\,\varphi(2r)\,(kT)^{2r}]\,, \end{aligned}$$

r is the set of real integers and $\varphi(2r)$ is the zeta function of order $2r$.

For $\alpha \to 0$, (4) takes the well-known form [7]

$$n_0=N_c\,\theta\sum_{n=0}^{n_{\max}}F_{-1/2}(\eta)\,, \tag{5}$$

where $N_c \equiv 2\,(2\pi\,m_d\,kT/h^2)^{3/2}$, $m_d \equiv (m_1\,m_2\,m_3)^{1/3}$, $\theta \equiv \hbar\,\omega_0/kT$, $\omega_0 \equiv e\,B/m_1\,m_2$, $\eta \equiv [E_F-(n+\tfrac{1}{2})\,\hbar\,\omega_0]/kT$ and $F_j(\eta)$ is the one-parameter Fermi-Dirac integral of order j as defined by Blakemore [7].

Using (2) and (4) and taking the parameters [8] $m_1=m_0/172$, $m_2=m_0/0.8$, $m_3=m_0/88.5$, $E_g=0.0153$ eV and $m_2 \approx m_2'$ we have plotted $m_n^*(E_F)/m_3$ versus $1/B$ for

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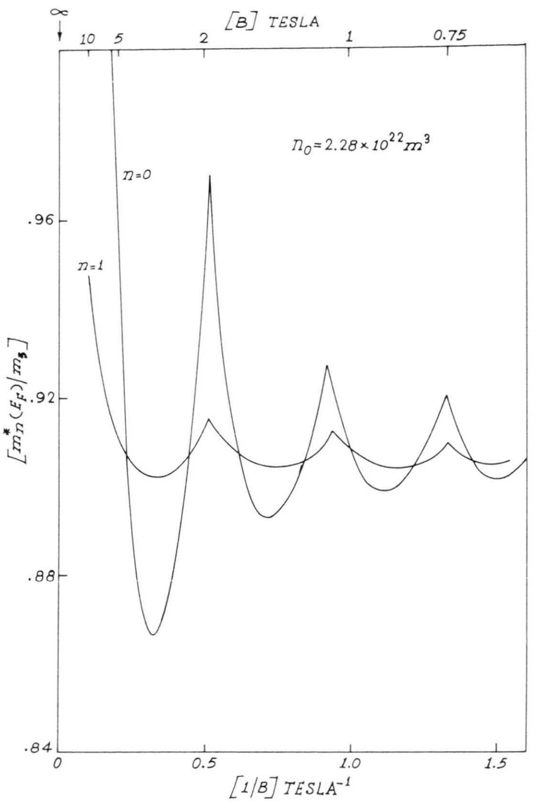

Fig. 1. Plot of the $m_n^*(E_F)/m_3$ for the electrons in bismuth versus $1/B$ of the first two magnetic sub-bands at low temperatures.

the electrons in Bi of the first two magnetic subbands at very low temperatures as shown in Fig. 1 corresponding to an electron concentration of $2.28 \times 10^{22}\,m^{-3}$. It appears from the figure that the magnetic quantization increases the effective electron mass along the direction of the magnetic field, the index-dependent behaviour of the same mass under magnetic quantization in Bi being solely due to band-nonparabolicity, which can be demonstrated by comparing (2) and (3), and is a special feature of the MeClure and Choi model. Incidentally the SdH period, being given by $\Delta(1/B) = 2e(3\pi^2)^{-2/3}(\hbar n_0^{2/3})^{-1}$, is related only to the carrier concentration in the sample and does not depend on the other physical parameters as stated in [9]. Besides, the SdH period as determined graphically has been found to be 0.41 Tesla^{-1}, which is in good agreement with the value 0.3945 Tesla^{-1} determined theoretically. Moreover, the contribution of the oscillatory index dependent effective mass on the oscillatory mobility would be increasingly significant. It may also be noted that at very low temperatures in Bi, the broadening of the Landau levels due to collisional effects may be neglected and that the effects of spin-splitting would simply be to increase the number of spikes of oscillation and to reduce their amplitudes. Though in a more rigorous treatment the many-body effects should be considered along with a self-consistent procedure, this simplified analysis exhibits the basic qualitative features of the index dependent oscillatory effective electron mass in Bi under magnetic quantization with reasonable accuracy.

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Verantwortlich für den Inhalt: A. KLEMM
Satz und Druck: Konrad Triltsch, Würzburg